A New Method For Solving Potential Problems Which Contain Straight Or Curved Strip Conductors **G-13**

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A new method for analyzing micro strip transmission lines is presented. This method uses the superposition of Green's function on a Riemann surface[1] and developed from research into the Charge Simulation Method[2]. By using this method , we can accurately calculate the potential field around straight or curved strip conductors which are essential in microwave transmission lines. The remarkable features of this method are

1 an accuracy of 10^{-5} is easily achieved
2 the accuracy near the tip of strip conductors is the same as that of an ordinary point
3 this method is applicable to problems which contain many straight or curved strip conductors
4 the error estimation is easy and
5 the computing time is very short.

As examples, some microwave transmission lines in a homogeneous medium are analyzed in TEM mode.

1 Introduction

The charge simulation method[2](we will call this CSM hereafter) is a superposition method of Green's function. It corresponds to the boundary method and collocation method in terms of the Method of Weighted Residuals. The CSM is not considered to be straight-forward, because contour points and source points must be determined by trial and error. However the estimation of error in the CSM is very simple, because only the value on the boundary need be compared with the boundary conditionsince it is here the maximum error appears. A high degree of accuracy can be easily obtained if the shape of the boundary concerned is not too complicated. Its evident and essential defect is that error becomes great for the exterior problem of very thin region, because of the singularity of the Green's function. To overcome this difficulty , a Riemann surface associated with the two-valued complex transformation $z=(t+1/t)/2$ is used. The shape of the branch cut is chosen such that it coincides with the shape of the strip conductor in transmission lines. The poles of the Green's functions are placed on one of the two sheets of Riemann surface and their influence on the other sheet is superposed similarly to the ordinary CSM. By using this complex transformation , the curved arc without area is transformed into a closed line with wide area. We can then keep the poles of Green's function apart from the boundary. This fact enables us to reduce the influence of the singularity of Green's function. This method is suitable for solving the exterior potential problems of very narrow regions like strip conductors.

So far a number of semi analytical and numerical methods have been applied in the analysis of micro strip transmission lines. These does not seem to be an effective way of accurately solving the problems which contain strip conductors of arbitrary shape.

2 Green's Functions On A Riemann Surface

At first, We would like to explain the complex transformation $z=(t+1/t)/2$ which is essential in this paper. By simple reduction , we obtain the inverse transformation

$$t = z + \sqrt{z^2-1} \qquad (1)$$

$$t = z - \sqrt{z^2-1} \qquad (2)$$

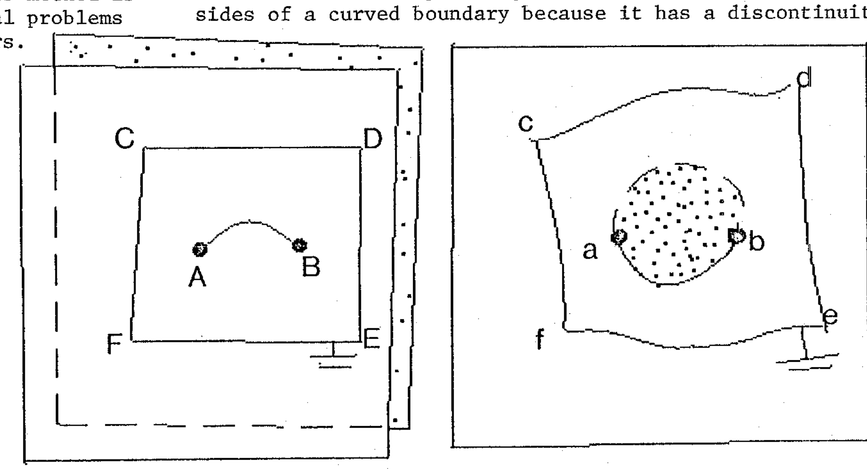
where $\sqrt{z^2-1}$ is the complex function that takes the value $\sqrt{3}$ at z=2 . The Riemann surface that we need consists of two z-planes as shown in Fig.1(a). The branch cut(curved line) which connects two points A(-1,0) and B(1,0) is transformed into a closed line on the t-plane as depicted in Fig.1(b) by above transformation equations. One of the two sheets of the Riemann surface is transformed into the outer region of the closed line by eq.(1) and the other sheet is transformed into the inner region of the closed line by eq.(2). We designate the former sheet as sheet 1 and the latter one as sheet 2, respectively. The Green's function on the Riemann surface is the image of the ordinary Green's function on the t-plane

$$G(t;t_k) = 1/2\pi \log(t-t_k) \qquad (3)$$

where t_k means the pole of the Greens function and t is the potential point. The image of eq.(3) on z-plane is

$$G(z;z_k) = 1/2\pi\log(z+\sqrt{z^2-1} -z_k+\sqrt{z_k^2-1}), \qquad (4)$$

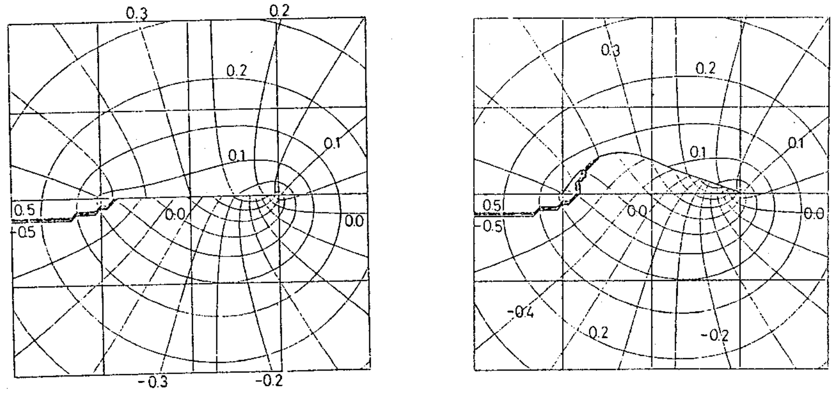
where we assumed that z and z_k are placed on sheet 1 and sheet 2 , respectively. In Fig.2 , the contour maps of the real and imaginary parts of eq.(4) are shown. Fig.2(a) and (b) correspond to the case of straight and curved cut, respectively. As one can see easily from these pictures , this Green's function has nosingularity around the point z_k hence the poles of this Green's function can be place anywhere. Also, these functions are suitable for expressing different values at both sides of a curved boundary because it has a discontinuity



(a) z-planes(Riemann surface) (b) t-plane

Fig.1 Complex Transformation $z = (t + 1/t)/2$

along the branch cut. This two-valued complex function is not straightforward to calculate. A simple way for calculating these Green's functions was given by the author[1].



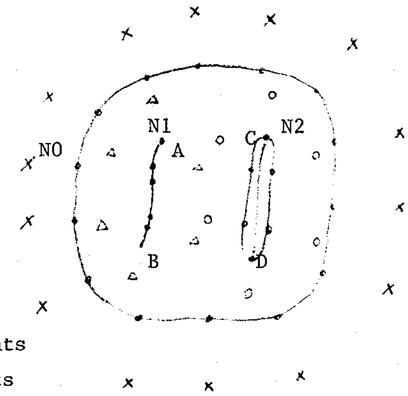
(a) Straight cut (b) Curved cut

Fig2 Green's function on a Riemann surface

3 How To Use Green's Function On Riemann Surface

Here we consider a Dirichlet problem of Laplace's equation in domain D which containes two very narrow regions as its boundary as shown in Fig.3. The shape of one inner boundary is very narrow and almost line. The other inner boundary is also narrow and contains some area inside it. But it has not enough area to yield good accuracy by ordinary CSM. The prescribed boundary value on the outer boundary is V0 , and the values on the inner boundaries are V1 and V2 , respectively. In Fig.3 , charges are represented by three kinds of notations, i.e., x , △ and o . Their meanings are given by

x :Green's function on the ordinary plane
△ :Green's function on the Riemann surface with two branch points A and B
o :Green's function on the Riemann surface with two branch points C and D.



• : contour points
x △ o : source points

Fig.3 How to use Green's functions

The number of these three kinds of Green's functions, i.e., the number of charges placed near each boundary, are N0 , N1 and N2 , respectively. The shapes of branch cuts of above two different Riemann surface have to coincide with the corresponding curved lines which connects A and B or C and D as shown in Fig.3. The solution to this problem is assumed by the following formulae

$$\phi(x,y) = \sum_{k=1}^{N} Q_k \, \mathrm{Re}\{G(z;z_k)\}, \tag{5}$$

where N=N0+N1+N2 and

$$G(z;Z_k) = 1/2\pi \begin{cases} \log(z-z_k) & 0<k\leq N0 \\ \log\{z+\sqrt{(z-A)(z-B)}-z_k+\sqrt{(z_k-A)(z_k-B)}\}, & N0<k\leq N0+N1 \\ \log\{z+\sqrt{(z-C)(z-D)}-z_k+\sqrt{(z_k-C)(z_k-D)}\}, & N0+N1<k\leq N. \end{cases} \tag{6}$$

The unknown constants Q_k are determined in a way similar to the ordinary CSM by imposing the boundary condition at selected collocation points. It is obvious that this technique is applicable to the problem in a region that contains many narrow boundaries of arbitrary shape.

4 On The Error Estimation in CSM

The remarkable feature of the CSM is that[3] the maximum value of absolute error can be easily obtaind simply by comparing the computed value with the boundary condition because of maximum value principle of error. In Fig.4 , a typical contour map of error distribution in CSM solution for a parallel condenser is shown. In this figure the number +6 and -5 associated with contour lines denote errors of $+10^{-6}$ and -10^{-5}, respectively. In this analysis the boundary condition at center line(y=0) of two parallel electrodes is satisfied analytically by using the image method. Hence the only boundary on which we have to impose conditions is the

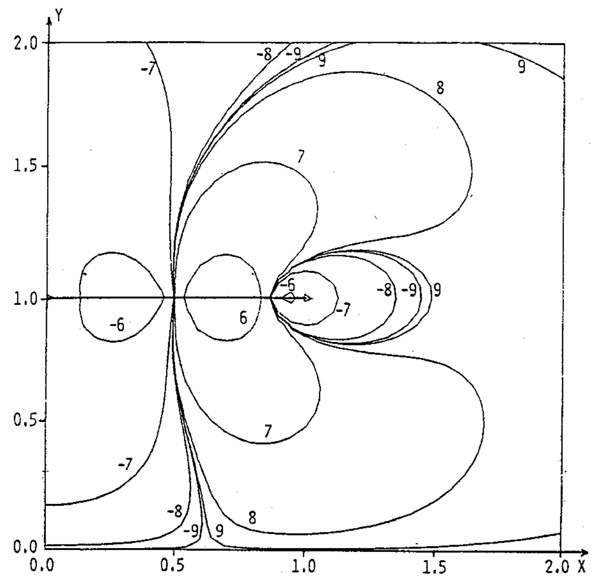


Fig.4 A typical error distribution in CSM solution for parallel condenser. The straight line which connects points(0,1) and (1,1) is electrode.

electrode of condenser. One can easily recognize that the maximum absolute error appears on this electrode.

Usually the error becomes large at the middle of adjacent contour points so we take the following value as the measure of average error

$$F = \sqrt{\sum_{k=1}^{N} E_k^2 \,/\, N}\,, \tag{7}$$

where E_k is the error at the middle point between the k and k+1 th contour points.

5 Numerical Examples

5.1 Cylindrical-coaxial strip transmission line

In Fig.5, a cross-sectional view of the cylindrical-coaxial strip transmission line with x-y coordinate system is shown. The straight strip of width 2 is located inside a shielded cylinder of radius 2. The strip in a homogeneous dielectric of permittivity ε_0 is charged to ϕ=1v. The contour points on the strip are determined in the t-plane by dividing the unit circle into N1 equal parts where N1 is the number of contour points placed on the strip(the same as the number of charge points on the sheet 2 of Riemann surface). When this unit circle contracts by the ratio R1 , we designate the position to which the contour points move as charge points. In the practical calculation , all data are given by the value on the z-plane. Hence, contour points have values on the straight line which connects the point (-1,0) and (1,0) while charge points are pleced on a elliptic circle with foci of(-1,0) and (1,0). The contour points on the outer cylinder are determined by dividing the outer circle into N2 equal parts and so on.

The characteristic impedance of microstrip transmission line in a homogeneous dielectric is evaluated from the electric charge on the strip conductor. It is easily shown that this value is equal to the summation of charges Q_k on Riemann surface. The line capacitance CO is

$$CO = \sum_{k=1}^{N1} Q_k / \phi \quad , \tag{8}$$

and the characteristic impedance ZO is

$$Zo = 1/CO/VO \quad , \tag{9}$$

where VO is the velocity of light in free space. When N1 =16,N2=24,R1=0.1 and R2=0.5, the capacitance ε_0CO = 4.558728405900 is obtained. The accuracy of this value is higher than 10^{-10}.

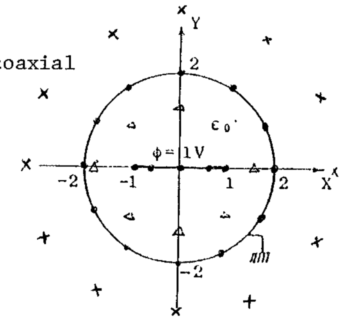
5.2 Rectangular-coaxial strip transmission line[4]

Fig.6 shows the cross-section of the rectangular-coaxial transmission line and its characteristic impedance ZO against W/b for a/W=2. The arrangement of contour and charge points near and on the strip is the same as that in the previous case. Table 1 shows a comparison of the calculated values of ZO for several a/b and W/b and those obtained by Tippet and Chang using a conformal maping method.

5.3 Cylindrical-coaxial strip transmission line[5] with a circular arc strip

This technique is also applicable for curved strips. In Fig.7, the cross-section of a cylindrical-coaxial line with a circularly curved strip and its characteristic impedance ZO for various geometries are shown. In the case of b/a=2 and c/a=1.5 , the calculated values of ZO are 18.422, 26.479 and 47.075 for α= 90, 60 and 30, respectively.

Fig.5 Cylindrical coaxial strip transmission line



× △: source points
• : contour points

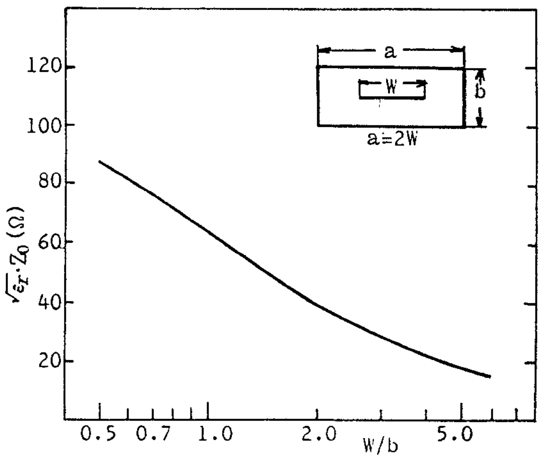


Fig.6 Cross-section of the rectangular-coaxial transmission line and its characteristic impedance

Table I Comparisons of calculated results

Case		Present		Tippet & Chang(4)
a/b	w/b	$Z_0(\Omega)$	Average potential error (%)	$Z_0(\Omega)$
1	0.80	54.636	0.034	54.54
1	0.50	87.038	0.014	87.03
2	0.50	99.841	0.008	99.82
2	1.00	64.104	0.012	64.10

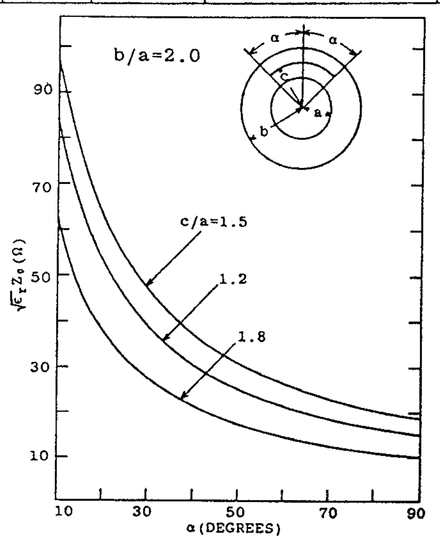


Fig.7 Cross-section of the cylindrical-coaxial transmission line and its characteristic impedance.

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